OPTIMAL DESIGN OF TEE-BEAM —A LIMIT STATE APPROACH

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By

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TO

MY PARENTS

CERTIFICATE

Certified that the work presented in this thesis entitled "Optimal Design of Tee Beam - A Limit State Approach" by Mr. Siva Prasad Darbhamulla has been carried out under my supervision and it has not been submitted elsewhere for a degree.

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ABSTRACT

The objective of the present investigation was to arrive at an optimal design of the basic structural element namely the Tee beam. The limit state theory, a recent design method was adopted. The optimization was carried out for the minimum cost. A nonlinear programming method with interior penalty function was utilized . A great deal of emphasis was given to the serviceability conditions like deflection and crack widths.

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LIST OF SYMBOLS

a	depth of stress block in Whitney's method
a_{cr}	distance between the reinforcement bar and a point on surface of the beam
A_{sc}	area of compressive steel
A_{st}	area of tensile steel
A_{sv}	area of shear reinforcement
b	width of the rib of the Tee beam
b_t	width of the rib at the level of the center of gravity of tension steel
B	width of the flange of the Tee beam
c_{st}	center to center spacing of the main steel in the slab
c_v	center to center spacing of the stirrups in the Tee beam
d	effective depth of the Tee beam
d_c	effective cover of concrete to the compressive steel
d_n	depth of neutral axis in Tee beam
d_s	effective cover of concrete to the tension steel
D	overall depth of the Tee beam
$\vec{D}$	design vector

$\vec{D}_o$	initial design vector
E_c	short-term modulus of concrete
E_{ce}	long-term modulus of concrete
E_{cs}	shrinkage modulus of concrete
E_s	modulus of elasticity of steel
f_c	cylinder (300 mm ∅, 600 mm long) crushing strength
f_{cu}	characteristic cube (150 mm sides) strength
f_y	ultimate yield stress of tensile steel
f_{yc}	creep stress in steel
f_{yt}	first yield stress of steel
F_c	total compressive force
F_s	total tensile force
$F(\vec{D})$	objective function for minimization
g_j	constraints in the formulated problem
G_i	function of constraints used to form the unconstrained problem
I_e	effective moment of inertia of the section
I_g	moment of inertia of gross section neglecting the reinforcement
I_r	moment of inertia of the cracked section
k	constant with appropriate subscripts
K	stress block factors with appropriate subscripts

l_b	effective span of the beam
l_k	equality constraints
l_s	effective span of the slab
m	modular ratio
M_B	moment caused by external loads on the beam
M_r	moment of resistance
M_{rB}	moment of resistance of the beam
M_{rs}	moment of resistance of the slab
M_s	moment caused by the external loads on the slab
P	probability
p	probability level
r_k	penalty parameter
s	strain with appropriate subscript
s_o	strain at the junction of parabola and rectangle in the rectangular parabolic stress block
s_{cs}	shrinkage strain in concrete
s_{cu}	ultimate concrete strain
s_1	actual strain in steel
s_2	strain in steel at final yield
s_3	strain in steel at first yield
v	shear stress
v_b	actual bond stress
v_{ba}	allowable bond stress

V	shear force
W	loads with appropriate subscript
w_m	maximum surface crack width
w_s	surface crack width vertically below the steel bars
x	design variables
y	deflection with appropriate subscript
z	lever arms with appropriate subscript
$\bar{z}$	lever arm for the total bending forces of resistance
α	partial safety factors with appropriate subscript
β	a constant used in the analysis of deflection due to shrinkage
γ_s	unit weight of steel in N/mm^3
ϕ	curvature
ϕ_k	formulated unconstrained minimization function
ξ	a coefficient used in the calculation of the long-term deflection.

CHAPTER I

INTRODUCTION

1.1 GENERAL

The basic role of a structural engineer is to plan, design and construct a structure. A structural designer in particular should be equipped with the knowledge of loads, materials, design methodologies and the advances in computational techniques. Traditionally structural designs were evolved over a period of time. Specially during the II world war, the acute shortage of materials like steel made the designers in the war torn countries to take a fresh look at the reserve strength of the materials. This has led to the plastic design of structures. However, the advent of computers has revolutionized the hitherto unsolved problems of the early fifties and large structures can be designed effectively for minimum cost.

1.2 DESIGN PROCESSES

The design of a structure means the proportioning of various components according to the needs of the customer. A creative sense, scientific knowledge and a little practical experience when combined, produce a satisfactory

solution. A thorough knowledge of the properties of materials are as important as the methods of structural analysis. Different idealizations of the material properties describe different design methods. The object of structural design is to obtain a structural solution which gives the greatest overall economy by providing the optimum while satisfying all the requirements of the structure. It is clear that the structural costs rise rapidly if other requirements become too onerous. So, a method which is simple, rational and readily applicable should be the aim of a designer. To understand the general design tools available at present, a brief description is given in the following paragraphs.

1.2.1 ELASTIC DESIGN (1)

This is the most widely used traditional and well established design tool. It assumes the elastic behaviour of the structure associated with the assumption that the materials used individually and as a composite usually obey the Hooke's law. A structure is generally designed for a particular case as flexure and other aspects are checked so that the design is feasible. In this method, the factor of safety is ensured by limiting the maximum stresses under working loads to a fraction of the values determined

by experiments. Composite materials are transformed as unitary materials by using the modular ratio of the materials. The section may thus be transformed into a homogeneous one **for** the purposes of analysis. Unfortunately, the actual factor of safety i.e. the ratio between the collapse load and the working load, is different for different structures and loading conditions. Hence this design method is not rational so far as the actual factor of safety is concerned. However, this method is handy for studying the behaviour of structure under working loads.

1.2.2 ULTIMATE LOAD DESIGN

This method is rational when compared with the elastic method described earlier. In this method, the ultimate values such as ultimate moment of capacity, ultimate shear capacity etc. are computed. The stress-strain curves for concrete and steel are idealized prior to their application in the design process. For example, the actual stress block in compression zone of a concrete section is idealized as a rectangular stress block although the actual stress-strain diagram is rectangular parabola which inturn is a better approximation of the recorded stress-strain curve of concrete in compression. The ultimate strain in concrete is assumed to be 0.003 for

all grades. The steel stress-strain curve is idealized as a bilinear curve. The slope suddenly changes to zero at the yield point. In this method,load factors are used to arrive at an ultimate load from the given working loads. Various types of loads (dead load, live load, wind load etc.) have different load factors. In the design of beams, usually the section is proportioned for flexure. This method, however does not give any idea about the behaviour of the structure at working loads.

1.2.3 LIMIT STATE DESIGN

The recent tool of design available to an engineer is the 'Limit state design'. It is more rational and an advanced design method wherein more accurate stress-strain curves are used in the design of the structure. The term limit state is defined in the following chapter. The advantage of this method is that the limit state can be choosen according to the needs of the customer. It is also fairly easy to apply this method for ordinary design problems. For the first time in literature a detailed design for the serviceability requirements like deflections, cracks, local damages etc. are considered in this method of design. Recently, codes have also started adopting this as the standard design. This method of design adopts both

the elastic and ultimate theories that weredescribed earlier. More accurate determination of design loads is possible with the introduction of the partial safety factors. All this has made the author to adopt this process of design in the present thesis. Although this is byfar the best design method, it does not yield unique solutions.

1.2.4 PROBABILISTIC DESIGN

In nature, the material properties and the environmental forces are not deterministic. But in the foregoing design methods, the loads and material properties are considered as deterministic. The purpose of any design is the achievement of the probabilities that the structure is fit for use under all the circumstances and loading conditions. It is evident that any general design requires some quantities of the design variables obtained by experimentation. They are found out applying the statistical methods. But in practice the strength varies. Such a design incorporating the variability of materials and loads may be termed as 'Probabilistic design'. This branch of science has caught the attention of many researchers around the globe. The basic design process hinges on the decision of the engineer as to how much level of probability is to be allowed for in the design. The general design

equation looks as follows.

$$P\left[f\,(\vec{D})\right] \leq p \qquad (1.1)$$

which is read as the probability (P) of a design function $(f(\vec{D}))$ is at most limited to the probability level (p).

1.2.5 OPTIMAL DESIGN

The designs that are arrived at are nonunique.Hence some design are bound to be uneconomical in terms of material consumption, cost etc. Thus it is imperative for optimizing a given design process in a systematic way. The optimal design may be in any domain of the material behaviour as elastic,plastic, visco-elastic etc. This is achieved either by using mathematical programming or optimal synthasis tools.

1.3 OPTIMIZATION NEEDS

In the case of aerospace vehicles minimum weight is the key factor whereas in developing countries like India, there should be a consceintious effort on the part of a designer to see that the structure consumes not only the minimum quantities of materials but also the overall the cost also should be minimum. The disproportionate increase in costs of building materials will make an engineer think

about the design adopted. Large structures (multi-storied buildings, dams etc.) can now be designed in an optimal manner, because of the advent of the fast digital computers.

An optimal design problem has an objective function and certain constraints which are expressed as functions of design variables.

1.4 LITERATURE SURVEY

The elastic theory of structural design being conservative, the need for rational and less conservative methods of design was felt by structural engineers as early as 1937. The concepts of ultimate load theory were developed by assuming different material properties to suit their convenience. One of the researchers that had developed the theories, C.S. Whitney was the most successful one. For the first time he had proposed the design by the consideration of a stress block in the compressive zone of concrete. He had assumed a simplified shape of the stress block which was rectangular (1). Later researchers had conducted experiments and different propositions for stress blocks were given. During that period engineers had not taken care the serviceability conditions. They had proposed designs for collapse of the structure. Researchers had

started thinking about the analysis for serviceability and local damages around 1965.

Bengt B. Broms (2) conducted experiments on short tension members and developed a simple method for calculation of crack width and crack spacing in R.C. members. It was felt afterwards that a rigorous analysis method had to be propsed, unfortunately no researcher could give exact determination of crack widths as the nature of the cracks is irregular. Then people had started to control the cracks by same means. The American Concrete Institute Committee submitted a report (3) in 1972 on the control of cracks. It covered the basic mechanism of microcracking and fraction mechanics in concrete. This report had made a mention of the control of cracking due to shrinkage and flexure . It had also reported the long-term effects on cracking. In this report a recommendation for construction was also provided in order to control the cracks. Gergely (4) had studied the effects of distribution of reinforcement for the control of crack. Later researchers Nawy (5), Orenstein (6) had suggested methods for control of cracks in two-way slabs. Elveny (7) had studied the behaviour of R.C. member under normal conditions of drying to predict shrinkage stresses and deformations. Theoretical determination of curvatures were suggested. Deflection of

R.C. flexural members was also studied by Branson (8) and principal factors that effect the short-term and the long-term deflections were discussed. Alcock and Pauw (9) had proposed design methods for controlled-deflection design for R.C. beams and slabs. Keeping all this in view Hughes(1) had given detailed description of the limit state theory and proposed certain design methods. MacGinley in his book (10) presented design theory and worked a number of examples based upon the British code (11) which had adopted the limit state design method for the first time. The Indian Standard Institution is also bringing out a code of practice based upon the limit state theory (13).

1.5 SCOPE OF THE THESIS

In this investigation, the optimal design of Tee beam has been taken to develop ready reckoners for obtaining near optimal preliminary design for office floors. Definitions, of all of the terms used, are presented in Chapter 2. A brief mention of the method is also given in the same chapter. The design applied to a particular case of Tee beam is presented in Chapter 3. The advanced method of design named the limit state method is applied. The optimization is done for cost to get economical designs of Tee beams. The Tee beam is selected as the element for

design with a view that it is the most widely adopted structural element.

The results and discussions are presented in Chapter 5 and an overview is presented in Chapter 6.

CHAPTER II

DEFINITIONS AND METHODOLOGY

2.1 GENERAL

Every scientific process has certain basic terms which are to be defined. For the sake of completeness definitions of the terms used in the present work are given herein. It is also necessary to give a brief account of the methods applied.

2.2 DEFINITIONS

2.2.1 STRUCTURE

The skeleton of frame work, with the primary elements as beams, slabs, columns and walls, to carry the permanent and superimposed loads safely. It transmits the total load to the foundation.

2.2.2 STRUCTURAL ELEMENTS

(1) Beams: Horizontal members carrying roof and floor slabs. They resist loads in bending, shear and bond and may be simply supported, continuous or cantilevered.

(2) Slabs: Horizontal plate elements which carry floor and roof loads. They may be simply supported or continous.

2.2.3 SUPPORT CONDITIONS

(1) Simply supported: Mechanical hinges which do not resist any moment, transfering the loads to the supporting members.

(2) Continous: Members which are continuous over the supports have these support conditions.

(3) Fixed: The support conditions wherein the member is built into the support. An example of this is a cantilever beam.

2.2.4 MATERIALS AND PROPERTIES

(1) Bond: The adhesion of concrete to reinforcement.

(2) Reinforced concrete: The concrete along with a reinforcing material acts as a composite material. Used in the constructions of framed and walled buildings, bridges, culverts etc.

(3) Creep: Strain or plastic flow that occurs in concrete due to a sustained load which is expressed as a creep coefficient ($\emptyset$) in case of concrete.

(4) Cube strength: Crushing strength of a cube of 150 mm sides, at 28 days.

(5) Shrinkage: Contraction of concrete due to temperature changes. This is irreversible and results in cracking of the concrete.

(6) Strain(s): The change in length per unit length of a member. It can be caused by physical influences shrinkage etc.

(7) Stress-strain curve: A plot of stress against strain which shows the behaviour of the material under physical influences (loads).

(8) Yield Strength(f_y): The limit of elastic region, beyond which there is a rapid increase in strain for a small incremental load and or for no increase in load.

(9) Characteristic cube strength (f_{cu}): The strengths of materials below which not more than 5 percent of test results fall.

(10) Young's modulus (E): The ratio of stress to strain for a linear elastic material.

2.2.5 COMPONENTS OF A TEE BEAM

(1) Tee beam: A rectangular beam which is cast monolithic with the slab over it. It shapes out into a 'T'.

(2) Flange: The portion of the slab whose width (B) is in effect to act monolithically with the rectangular rib below it.

(3) Rib: The rectangular beam lying underneath the slab, whose width is denoted as 'b'.

(4) Depth (D) : The total depth of the rib and the slab put together is called the depth of the Tee beam.

(5) Effective depth (d): The depth measured from the top of flange to centre of gravity of the tension steel in the beam.

(6) Effective cover: The amount of cover of concrete measured as the depth of concrete from the nearest face to the centre of gravity of the tension steel (A_{st}) which is denoted as d_s and as d_c in case of compressive steel in the beam.

(7) Span: The centre to centre distance between the supports of the slab (l_s), resting on the beams or of the beam (l_b), resting on the walls or columns.

2.2.6 LIMIT STATE METHOD

(1) Limit state: State at which a structure becomes unfit for use. The main limit states are the limit states of collapse and serviceability.

(2) Characteristic loads: The working or service loads, classified into dead and imposed loads. They have a low probability of being exceeded during the life time of the structure.

(3) Design loads: The loads obtained, from the characteristic loads, by multiplying the same by the partial

safety factors of safety for loads.

(4) Design strength: The characteristic strengths divided by the partial safety factors of safety for materials.

(5) Partial factors of safety (α): Factors applied to loads and materials to obtain the design values. They allow for uncertainities in the estimation of loads and variations in the strengths of materials. These control the safety of structures.

(6) Ultimate strength of sections: A value based on the idealised stress-strain curves for the materials, taking account of the plastic behaviour.

(7) Modular ratio (m): The ratio of the Young's modulus of steel to that of concrete.

(8) Analysis: The process of determining the design actions, that are the shear, moment, bond etc. in the present work.

(9) Moment of resistance(M_r): The moment that can be safely resisted by a structural element.

(10) Deflection: The displacement of a member with respect to its original position. Generally measured with respect to the axis of the member.

(11) Crack width (W): The width of a crack formed in concrete due to shrinkage, creep or fracture of the structure.

(12) Collapse: The failure of a structural member.

(13) Short-term deflections (y_i): The deflection of a structure in a short duration.

(14) Long-term deflection (y_c, y_s): The deflection of a structure which are dependent on the time of loading. These are generally shrinkage and creep deflections.

(15) Spalling of concrete: The ripping out of concrete due to excessive deflections there by causing cracks in concrete.

(16) Dead loads: The loads that are permanent in nature contributed by the weight of the structural elements.

(17) Imposed loads: The loads which are not permanent and sometimes may be dynamic in nature.

(18) Moment of Inertia (I) : The second moment of area with respect to any axis. It is referred to an axis.

(19) Stress block: The idealised stress distribution, of concrete in compfession, above the neutral axis.

(20) Neutral axis: The axis where the strain in the cross-section is zero.

(21) Depth of neutral axis (d_n): The depth measured from the farthest fibre in compression to the neutral axis.

(22) Under-reinforced section: A section where the steel gets over stressed and therefore yields. The strain increases enormously without any increase in the stress thereby shifting the neutral axis towards the compression face. As a result the final failure of concrete occurs, which is a secondary effect. This failure is called a 'Tensile failure' or 'Secondary compression failure'.

(23) Balanced section: The one for which the steel yields as the concrete just reaches its ultimate strain. That is, the beam fails simultaneously in flexural compression and tension.

(24) Over-reinforced section: The steel remains elastic while the concrete strains without increasing its moment. So, the neutral axis falls while the concrete rapidly strains and crushing of concrete in compressive zone takes place.

(25) Curvature (ϕ) : Defined as the reciprocal of radius of curvature, which is the response of a structure due to the lateral loading that causes flexural stresses in the members.

2.2.7 OPTIMIZATION

(1) Optimization: The process of improving a merit

function to get the best solution to the given problem.

(2) Design variables(x_i): The quantities which are to be determined to satisfy the given norms of a design.

(3) Constraints (g_i): The restrictions on the functions of the design variables which influence the physical nature of the problem.

(4) Objective function (F): The function that has to be extremized (minimized or maximized) to get the best and most economical solution to a given problem. Some times this is also called a cost function or a merit function. This can also be stated as the criterion with respect to which the design is optimized. This is expressed in terms of the design variables.

2.3 METHODOLOGY

A summary of the different design methods available, for the design of a structure, are presented in the preceding chapter. The recnent method of design namely the 'Limit state design' is adopted in the present thesis. With facilities of the computer being available, the design is duly optimized for the total cost of the structure.

2.3.1 LIMIT STATE OF COLLAPSE

2.3.1.1 BENDING

The selected stress distributions are detailed in the next chapter. The stress block factors K_1, K_2 and K_3 are determined as (1).

$$K_1 \; K_3 = (27 + 0.35 \; f_c) \; / \; (22 + f_c) \qquad (2.1)$$

$$K_2 \quad = 0.5 - f_c/550 \qquad (2.2)$$

where f_c is the cylinder (300 mm dia, 600 mm long) crushing strength. The relation between the characteristic cube strength and the cylinder crushing strength is given according the equation (3.3).

Determination of the depth of neutral axis (d_n) is explained in article 3.3.1.2(a). After computing the depth of neutral axis the rectangular parabolic stress distribution (Fig. 3.3c) is used to determine the compressive force. The total compressive force is divided into two parts and is used for computations of the moment capacity of a section. A detailed account of this is given in the following chapter.

The bending moment (M) on the section by external loads is calculated as

$$M = k_2 \; w_u \; l^2 \qquad (2.3)$$

where k_2 is a factor dependent on the support conditions

of the structure. This thesis has incorporated a value of 0.1. W_u is the ultimate load computed as

$$W_u = \alpha_d W_d + \alpha_s W_s \qquad (2.4)$$

where α_d and α_s are the partial safety factors to dead load and imposed load respectively. In the present work α_d is taken as 1.4 and α_s as 1.6 as specified by unified code of practice (11).

2.3.1.2 SHEAR

The change in moment along the span of the beam causes shear on the cross-section of the beam. This in association with the bending stress induces diagonal cracks in the structure. Hence it is necessary to suitably design the structure taking care of the shear which in turn controls the diagonal cracking. The shear stress on the section is obtained in the same way as it is done in the case of an elastic method of design, except that the lever arm is replaced by the effective depth of the section. A section is also designed to see that the shear stresses caused or not too high and are limited to a maximum allowable shear stress (v_{all}) which is related to the characteristic strength of the concrete cube as

$$v_{all} = 0.6 f_{cu} + 1.6 \qquad (2.5)$$

Detailed analysis is given in the following chapter.

2.3.1.3 BOND

Bond is defined earlier as adhesion between the concrete and the reinforcing material. The bond stress analysis is done in the same way as it is done in the case of elastic theory. The bond stress varies along the length of reinforcement but for all practical considerations it is assumed to be constant. Bond in reinforcing concrete plays two different roles. Firstly, it provides the required anchorage to the reinforcement so that total bending stress is transfered to the reinforcement over a required length and secondly the average bond (v_b) requires sufficient amount of perimeter of the reinforcing material such that there is no slip of the reinforcement. The latter stress is called the average bond stress. The Tee beam does not have the problem of anchorage length except where there is a negative moment and curtailment of the main reinforcement.

The average bond stress allowed in concrete is dependent on the grade of concrete and the type of reinforcing bar used. Where plain bars are used without deformations or ribs over it then the allowable average bond stress (v_{ba}) can be calculated by the expression.

$$v_{ba} = 0.04\ f_{cu} + 0.40 \quad \text{for } f_{cu} \leq 25\ \text{N/mm}^2$$
$$= 0.04\ f_{cu} + 0.30 \quad \text{for } f_{cu} \leq 40\ \text{N/mm}^2$$
$$= 1.9 \quad \text{for } f_{cu} > 40\ \text{N/mm}^2 \quad (2.6)$$

In case, deformed bars are used these values given by equation (2.6) should be increased by 60 percent.

2.3.2 LIMIT STATE OF SERVICEABILITY

It is necessary that a structure should serve the function for which it is constructed. The functional utility will be lost if the structure has irregularities like large deflections, wider cracks, vibrations under working loads etc. In such a case users experience psychological fear for living in those dwellings. So, it is felt that a rigorous analysis of such secondary effects of the structure will enable an engineer to assess them properly and to limit them according to the needs of the customer. An analysis of the deflections and the cracks is done under this heading. The other serviceability conditions are out of scope of this thesis. The partial safety factors each of 1.0 are used to get the design loads.

2.3.2.1 DEFLECTION

The deflection of a reinforced concrete structure can be classified into two viz. short-term and long-term deflections. The response of a structure immediately

after application of a load is short term deflection. In the other case any sustained load will make a concrete structure creep. It is also evident that concrete has shrinkage effects thereby strain the concrete further. So, these two effects of creep and shrinkage make a reinforced concrete structure to deflect and such a deflection is termed as long-term deflection. The procedure to analyse the structure for estimating the short and long-term deflections is presented in the following articles.

2.3.2.1(a) SHORT-TERM DEFLECTION

As already mentioned reinforced concrete is not perfectly elastic. But for all practical conditions it can be safely assumed as an elastic material under the normal working loads. This assumption makes it simpler to analyse the structure easily and enables an engineer to apply the principles of elastic theory in the determination of deflections.

The general relationship relating the deflection of a structure to the moment at any section is (1) .

$$EI \quad \frac{d^2y}{dx^2} = - M \qquad (2.7)$$

where y is the deflection of the strudture,

M is the moment on the section and

EI is the flextural rigidity of the structure considered.

But the curvature of the structural member can be approximated as

$$\phi = \frac{-d^2 y}{dx^2} \qquad (2.8)$$

On substitution and rearranging the terms one gets

$$\phi = -\frac{M}{EI} \qquad (2.9)$$

The deflection y can be expressed, in terms of the curvature ϕ, and the length of the member l as

$$y = k\, l^2\, \phi \qquad (2.10)$$

where k = a constant given by the laws of mechanics applied to the structure.

The constant (k) can be easily calculated using the principles of mechanics or can be taken from standard tables which give its value for different boundary conditions. It is evident that one has to calculate the moment of inertia to compute the curvature and then the deflection of the structure. In the present analysis the moment of inertia (I_r) of cracked section and the moment of inertia (I_g) of the gross section are calculated to account for the stiffening effect of concrete in tension zone. The moment of inertia of the cracked section is

$$I_r = \frac{Bd_n^3}{3} + m\, A_{sc}\, (d_n - d_c)^2 + m\, A_{st} (d - d_n)^2 \qquad (2.11)$$

where d_n the depth of neutral axis is computed from the following equation

$$0.5\ Bd_n^2 + d_n((m-1)A_{sc} + mA_{st}) = mA_{st}d + (m-1)A_{sc}d_c \quad (2.12)$$

If the computed depth of neutral axis is greater than the depth of flange, then the depth of neutral axis is computed from equation (2.13) and correspondingly the moment of inertia of the section is obtained from equation (2.14).

$$0.5\ Bd_n^2 + d_n\left[(B-b)d_f + (m-1)A_{sc} + mA_{st}\right] = \frac{(B-b)}{2}d_f^2 + (m-1)A_{sc}\ d_c + mA_{st}d \quad (2.13)$$

$$I_r = \frac{bd_n^3}{3} + \frac{(B-b)d_f^3}{12} + (B-b)d_f(d_n - \frac{d_f}{2})^2 + mA_{sc}(d_n-d_c)^2 + mA_{st}(d-d_n)^2 \quad (2.14)$$

The gross moment of inertia is calculated using equations (2.15) through (2.18) with the depth of neutral axis being computed from equation (2.15) in case the depth of neutral axis is greater than the depth of the flange; if not, it is computed from the equation (2.17). The moment of inertia is calculated by one of the two equations (2.16) and (2.18).

$$d_n = \frac{bd^2}{2} / (Bd_f - bd_f + bD) \quad (2.15)$$

$$I_g = \frac{(B-b)d_f^3}{12} + (B-b)d_f\left(d_n - \frac{d_f}{2}\right)^2 + \frac{bd_n^3}{3} + \frac{b}{3}(D-d_n)^2 \qquad (2.16)$$

$$d_n = 0.5\ (bD^2 + Bd_f^2 - bd_f^2)\ /(bD + Bd_f - bd_f) \qquad (2.17)$$

$$I_g = \frac{B}{3}d_n^3 + \frac{b}{3}(D-d_n)^3 + \frac{(B-b)}{3}(d_f - d_n)^3 \qquad (2.18)$$

Then the effective moment of inertia (I_e) which encounterd the stiffering effect of concrete in tension zone is calculated as

$$I_e = \frac{I_r}{1.2 - \frac{M_r}{M} \cdot \frac{\bar{Z}}{D}\left(1 - \frac{d_n}{D}\right)\frac{b}{B}} \qquad (2.19)$$

such that $I_r \leq I_e \leq I_g$

where M_r = cracking moment calculated as

$$M_r = \frac{f_r\ I_g}{y_t} \qquad (2.20)$$

M = moment on the section considered.

$\bar{Z}$ = the lever arm

f_r = modulus of rupture of concrete which is related to the characteristic cube strength f_{cu} as

$$f_r = 0.7\sqrt{f_{cu}} \qquad (2.21)$$

y_t = distance between the centroidal axis, of the gross section, to the extreme tension fibre and can be

computed as

$$y_t = (D - d_n) \qquad (2.22)$$

where d_n is computed from equations (2.15) or (2.17) depending on whether the depth of neutral axis is greater or less than the depth of flange. Then the short-term deflection is obtained using equations (2.9) and (2.10) replacing the moment of inertia (I) by the effective moment of inertia (I_e) and the length (l) by the span (l_b) of the beam.

2.3.2.1(b) LONG-TERM DEFLECTION

The deflection due to shrinkage and creep of concrete are considered in the long-term deflection. The detailed analysis of these deflections are given in the article(3.3.2.1b).

2.3.2.2 CRACKS

Cracks in concrete in the tension zone of an ordinary reinforced structure are unavoidable. But in case of water retaining structures the design is done in such a way that there will not be any visible cracks. The width of cracks can not be obtained accurately, but some empirical formulae are given to calculate the width of cracks (10). The detailed procedure for the calculation of the crack widths and their limitations are given in the next chapter.

CHAPTER III

ANALYSIS OF REINFORCED CONCRETE TEE BEAM-LIMIT STATE APPROACH

3.1 GENERAL

A structure can be built as a framed structure or a walled structure. A framed structure has beams, columns and slabs as the primary design elements whereas a walled structure has walls, beams and slabs. The most common form of a beam is the Tee beam. A beam constructed monolithic with the slab over it will function as a Tee beam. It has increased area in compressive zone due to the monolithic action of the slab. All the codes of practice permit the design by the traditional and well established elastic theory or the ultimate load theory. Of late, the designs are based on the limit state theory. A design of the most commonly used structural element is herein presented. A brief summary of the properties of the materials is also given.

3.2 MATERIAL PROPERTIES

Concrete and steel are used as the materials of construction. The most widely adopted mix of concrete for the Tee beam and slab floors is of grade M20 which has a

characteristic cube strength (f_{cu}) of $20N/mm^2$. Hot-rolled mild steel with an ultimate yield stress of 250 N/mm^2 is considered in the design. Typical idealised stress-strain curves for concrete and steel are shown in Figs. (3.1) and (3.2).

3.2.1 CONCRETE

A rectangular parabolic stress block Fig. (3.3c) is used at the limit state in flexure for concrete in compression. The strain 's_o' in concrete at the junction of the parabola and rectangle is given by

$$s_o = \sqrt{f_{cu}}/5000 \qquad (3.1)$$

The ultimate strain s_{cu} in concrete is dependent on the grade of concrete used and is given by

$$s_{cu} = (4-f_c/45) \times 10^{-3} \qquad (3.2)$$

where f_c is the cylinder (300 mm dia and 600 mm long) crushing strength. The typical relation between the cylinder crushing strength and the characteristic cube strength is

$$f_c = 0.78\ f_{cu} \qquad (3.3)$$

The Young's modulus (E_c) of concrete is assumed to be independent of the stress level for a particular grade but

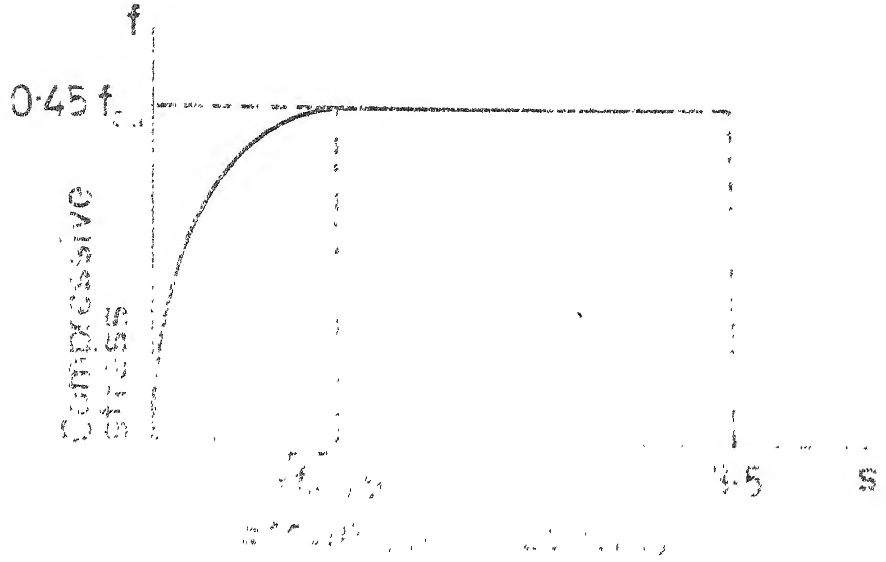

Fig.3·1 Design stress-strain curve for concrete
(f_{cu} in N/mm^2)

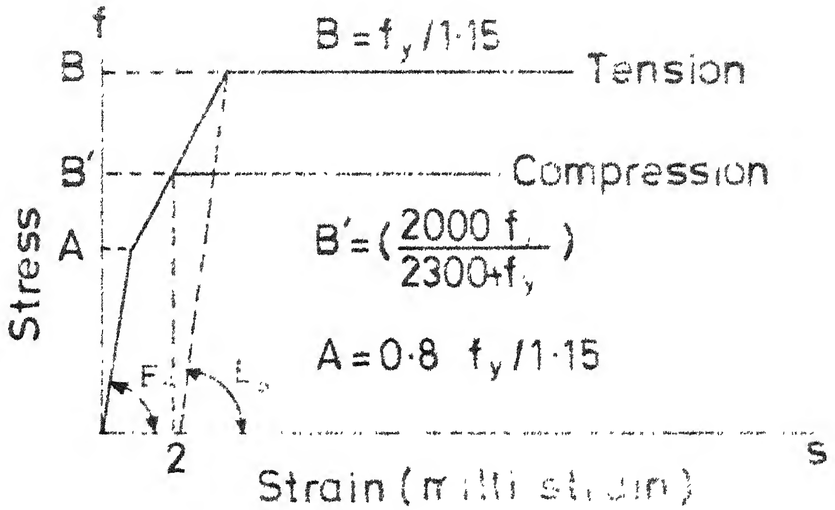

Fig.3·2 Design stress-strain curve for hot-rolled steel
(f_y in N/mm^2)

is dependent on the density (ς) of concrete. It is calculated as

$$E_c = 9.9\,\varsigma^2 \sqrt{f_{cu}} \qquad (3.4)$$

The long-term modulus is about half of the value given by the equation (3.4).

3.2.2 STEEL

A trilinear stress-strain relation is assumed in case of hot-rolled mild steel as shown in Fig. (3.2). Two different yield levels are assumed with the first yield occuring at a stress level of A and the second at B as indicated. Two different stress-strain relations are used wherein the second yield point B' in compression is at a lower level than that of tension value. The Young's modulus of steel (E_s) is assumed constant and same in compression and tension which is equal to 2.0×10^5 N/mm^2.

3.3 STRUCTURAL ANALYSIS

3.3.1 LIMIT STATE OF COLLAPSE

3.3.1.1 BASIC ASSUMPTIONS

The ultimate load capacity of the beam is computed based upon the following assumptions.

(1.) Plane sections remain plane even after bending.

(2) There is a perfect bond between concrete and steel.

(3) The stress distribution in the concrete in flexural compression is given by the stress-strain curve for concrete in uniaxial compression.

(4) The concrete is said to fail when the farthest fibre is strained to an ultimate value recorded in a direct compression test.

(5) The stress in the reinforcement is given by the appropriate stress-strain curve obtained in direct tension test.

(6) The tensile strength of concrete is ignored except that the stiffening effect of concrete in tension is considered while calculating the deflections of the beam.

3.3.1.2 FLEXURE

Consider a typical Tee beam section shown in Fig. (3.3). For design purposes it is convenient to assume a stress block shape which is as simple as possible. Many different shapes have been suggested from time to time. Some of them which are in use at present are shown in Fig. (3.3). The most widely used stress block is the rectangular stress block proposed by C.S. Whitney in 1937.

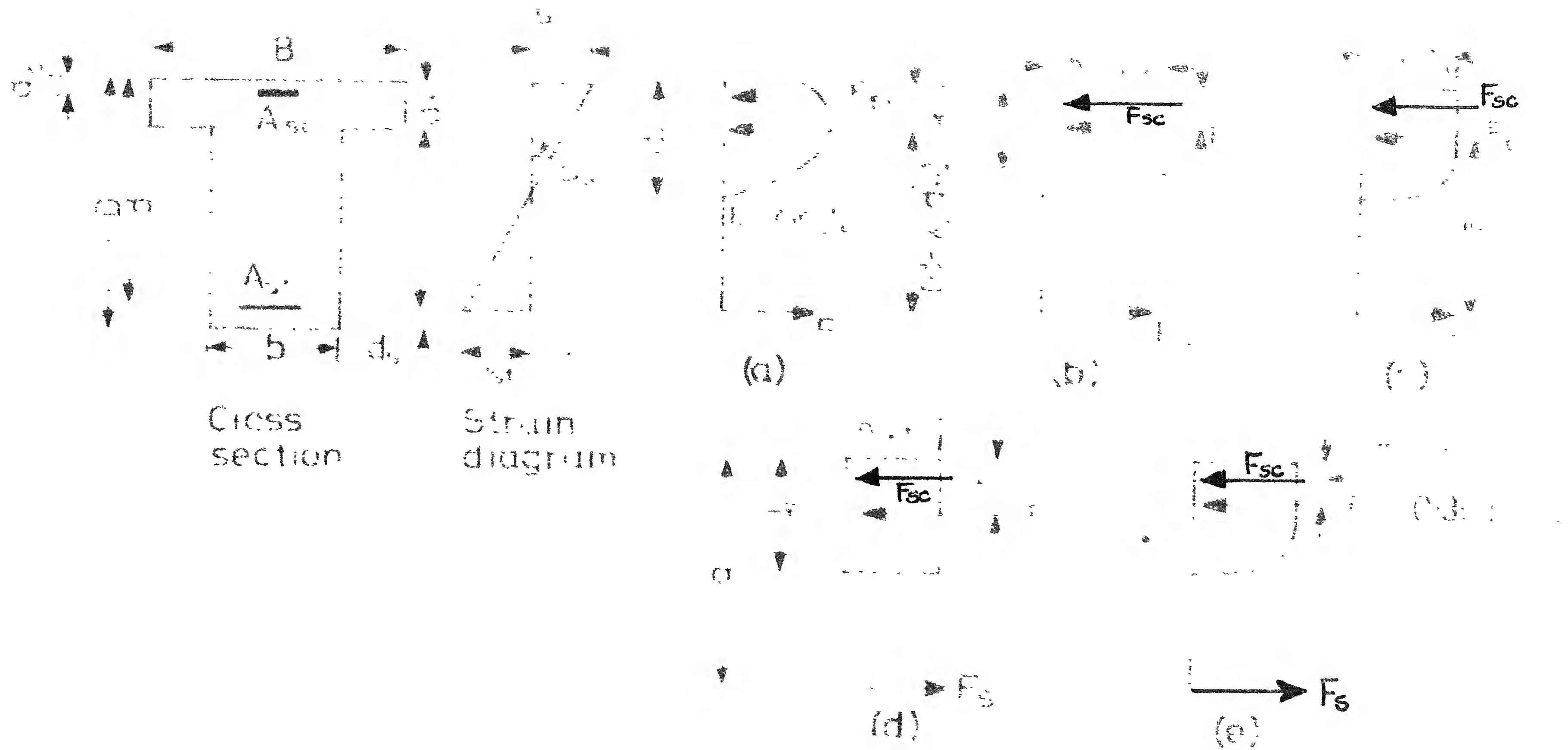

Fig. 3.3 Typical tee beam cross section – different stress blocks (a) Hogenstad (b) Whitney (c) Unified code (d) Simplified unified code (e) I.S draft code

In the present thesis the rectangular parabolic stress block shown in Fig. (3.3c) is adopted. The stress block factors (K_1, K_2 and K_3) are given by empirical formulae given in equations (2.1) and (2.2). The total compressive force F_c is

$$F_c = F_{c1} - F_{c2}$$

where F_{c1} is the total compressive force assuming an ordinary rectangular cross-section of breadth equal to the width of the flange (B) and F_{c2} is the compressive force under the flange projections, shown in Fig. (3.4a)

$$F_{c1} = K_1 K_3 \; f_c \; B \qquad (3.6)$$

$$F_{c2} \geq \frac{1}{2} \; E_c \; s_{c2} \; (B-b) \; (d-d_f) \qquad (3.7)$$

$$F_{c2} \leq \; K_1 K_3 \; f_c \; (B-b) \; (d-d_f) \qquad (3.8)$$

where s_{c2} is the strain in concrete at the soffit of the flange at failure, d is the effective depth of the section and d_f is the depth of flange.

3.3.1.2(a) Analysis of a beam is a trial and error process. Recommended step by step procedure is as follows:

(1) Determine the total compressive force F_c.

(2) Determine the total tensile force

$$F_s = f_{yt} \; A_{st} - A_{sc} \; f_{yc} \qquad (3.9)$$

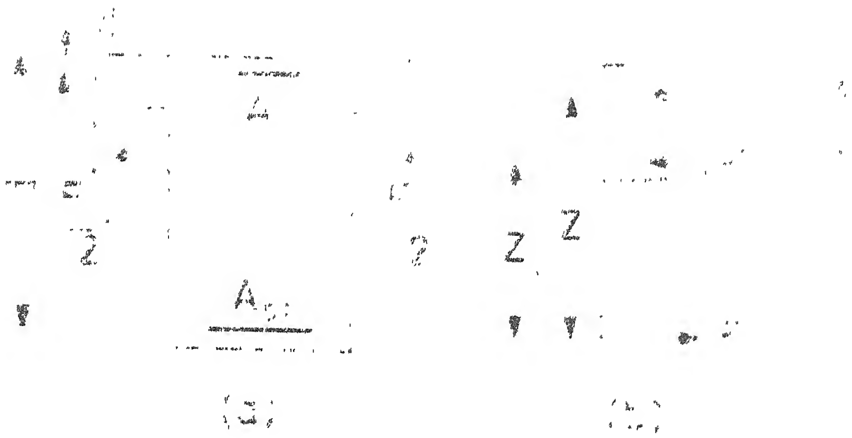

Fig.3·4 (a) Force in flange projections
(b) Forces and respective lever arms-rectangular parabolic stress block

where A_{st} and A_{sc} are the tensile and compressive reinforcement respectively.

(3) Determine the depth of neutral axis (d_{n1})

$$d_{n1} = F_s/F_c \qquad (3.10)$$

(4) If $d_{n1} < \frac{1}{2}$, the beam is under-reinforced. Then go to step 5. Go to step 6 if the beam is over-reinforced ($d_{n1} > \frac{1}{2}$ d). The section is balanced if $d_{n1} = \frac{1}{d}$ d.

(5) If the actual stress (f_s) in steel is greater than the first yield stress (f_{yt}), estimate the new depth of neutral axis

$$d_{n2}=(f_y/f_{yt})\ d_{n1} \qquad (3.11)$$

Then go to step 7.

(6) If f_s is less than f_{yt}, calculate f_s from the strain in steel (s_s)

$$s_s/s_{cu} = (d-d_n)/d_n \qquad (3.12)$$

where s_{cu} is the ultimate strain in concrete and

$$s_s=K\ d_n \qquad (3.13)$$

where $K= K_1K_3\ f_c\ b/A_{st}\ E_s$ (3.14)

On substituting equations (3.13) and (3.14) in equation(3.12), an equation in terms of d_n is obtained from which the new approximation for d_n is obtained.

$$(K/s_{cu})\ d_n^2 + d_n - d = 0 \qquad (3.15)$$

(7) Check if forces F_c and F_s balance; if not

$$d_{n3} = (F_s/F_c)\ d_{n2} \tag{3.16}$$

(i) when under-reinforced, see if the forces balance for d_{n3}.

(ii) If over-reinforced, d_{n3} over corrects, hence try the mean of d_{n2} and d_{n3}.

(8) If forces still do not balance, estimate subsequent d_n by interpolation technique.

(9) Once the forces balance, the depth of neutral axis is known from which the moment of resistance of the section is determined.

3.3.1.2(b) MOMENT OF RESISTANCE

Refering to Fig. (3.4b) the total compressive force is divided into two with F_{c1} being the resultant force of the rectangular stress block and F_{c2} being the resultant force of the parabolic portion of the stress block. The factor (k_1) for the depth of the parabolic stress block is calculated.

$$k_1 = 0.2\sqrt{f_{cu}}/3.5 \tag{3.17}$$

where f_{cu} is in N/mm^2

$$d_1 = k_1\ d_n \tag{3.18}$$

Hence the depth of the rectangular stress block (d_2) is

$$d_2 = d_n - d_1 \tag{3.19}$$

If the depth of neutral axis is greater than the thickness of flange stress block given by simplified unified code (1) is used for the sake of simplicity and is given in article 3.3.1.2(c).

Hence for a unit width of beam

$$F_{c1} = 0.45\ f_{cu}\ d_2 \tag{3.20}$$

$$F_{c2} = \frac{2}{3} \times 0.45\ f_{cu}\ d_1 \tag{3.21}$$

The lever arms Z_1, Z_2 to both the forces are determined from the geometry

$$Z_1 = (d - d_{2/2}) \tag{3.22}$$

$$Z_2 = (d - d_2 - \frac{3}{8}\ d_1) \tag{3.23}$$

Total compressive force (F_c) is determined as

$$F_c = F_{c1} + F_{c2} \tag{3.24}$$

Knowing the values of Z_1 and Z_2 the moment of resistance M_r is computed

$$M_r = F_{c1}\ Z_1 + F_{c2}\ Z_2 \tag{3.25}$$

3.3.1.2(c) As the depth of neutral axis is more than the thickness of the slab, the effective area of concrete in compression is the full flange and the portion of the web above the neutral axis. The rectangular parabolic stress

block poses a problem of geometry in the calculation of the total compressive force and the leverarm. So, a simpler and fairly accurate rectnagular stress block (1) is used , Fig. (3.3d)

The total compressive force F_c is divided into two forces, F_{c1} being the force in the flange and F_{c2} being the force in the portion of the web. Then

$$F_c = F_{c1} + F_{c2} \tag{3.26}$$

where

$$F_{c1} = 0.4\ f_{cu}\ B\ d_f \tag{3.27}$$

$$F_{c2} = 0.4\ f_{cu}\ b\ (d_n - d_f)$$

With these values of compressive forces the depth of neutral axis is obtained in the same way as described in article 3.3.1.2(a)

$$Z_1 = d_n - d_f/2 \tag{3.29}$$

$$Z_2 = d_n - \left(\frac{d_n + d_f}{2} \right) \tag{3.30}$$

Moment of resistance is calculated by using equation (3.25).

3.3.1.3 SHEAR

Unlike the elastic approach wherein it is assumed that uniform shear stresses act on vertical sections of

the beam. For the cracked section the situation is much more complicated. The shear is resisted by

(1) Uniform shear stresses on the compressive zone,

(2) aggregate interlock along the cracks.

(3) dowel action in the bars where the concrete between the cracks transmits shear force to the bars.

In view of these problems the shear stress calculated by simple formula similar to that given by elastic theory is retained

$$v = V/bd \tag{3.31}$$

The area of steel to resist these shear stresses is obtained

$$A_{sv} = \frac{b\ V c_v}{0.87\ f_y} \tag{3.32}$$

The spacings (s_v) of links is such that every potential crack is crossed by at least one link. To ensure this, the spacing is limited to 0.75 d in the direction of the span.

Shear stresses due to torsion are out of scope of this thesis as the member is not subjected to twist.

3.3.1.4 BOND

Bond is the frictional grip between steel and

concrete. Without proper bond, there is no composite material like reinforced concrete and the no slip condition can not be satisfied. Main emphasis is given to the local bond. The local bond is calculated in the same way as is in case of elastic approach.

$$f_{lb} = \frac{V}{\bar{Z}\ \Sigma o} \tag{3.33}$$

where f_{lb} is the actual local bond stress,

$\bar{Z}$ is the lever arm and

Σo is the sum of the perimeters of all the reinforcing bars.

3.3.2 LIMIT STATES OF SERVICEABILITY

3.3.2.1 DEFLECTION

The limit state of deflection is concerned with probable maximum loading under normal working conditions during the life time of the structure. Hence the characteristic loads are directly used with partial safety factors being 1.0 for dead load as well as superimposed load. The partial safety factor for materials is also 1.0.

The deflection of a structure has two components, the short-term deflection and the long-term deflection as defined earlier in Chapter II. The analysis for deflections

is based on the elastic methods and the convenient way is to find the curvature (ϕ) and then obtain the deflection(y).

$$\phi = \frac{M}{EI} \qquad (3.34)$$

where M is the moment due to external loads and

EI is the flexural rigidity of the beam.

$$y = k\, l_b^2\, \phi \qquad (3.35)$$

where k is a constant depending on the boundary conditions of the beam and l_b is the span of the beam.

The foregoing equation (3.35) for elastic deflection takes no account of variation in section properties along the span. The support and mid-span sections, for example, usually contain varying amount of reinforcement. However, the use of values for these terms based on the section at mid-span has been found to be satisfactory (10) for both simple and continuous beams.

3.3.2.1(a) SHORT-TERM DEFLECTIONS

Elastic theory assumes linear behaviour between stress and strain even at high loads. The concrete in reinforced members however, cracks at relatively low tensile stresses. As a result the load-deflection curve tends to be bilinear. The short-term deflection therefore should

be determined according to whether or not the member is cracked or uncracked. But with little sacrifice in accuracy the deflections are measured assuming a cracked section. The effect of stiffening of concrete in tension zone is considered as given earlier in the preceding chapter.

3.3.2.1(b) LONG-TERM DEFLECTIONS

The cause of long-term deflections, as described earlier, are mainly due to the effects of shrinkage and creep in the structure. Actual determination of the long-term deflections involves separate considerations of the actual shrinkage and creep coefficients of the concrete. However, if the expected shrinkage and creep strains are not excessive, then the total long-term deflection is assumed to be the sum of the short-term deflection due to all loads and the additional deflection due to creep and shrinkage.

The deflections (y_c) due to creep and shrinkage(y_s) are calculate as

$$y_s = \beta \, \Gamma_{cs} \, l_b^2 \qquad (3.36)$$

where β is a constant depending on the support conditions, Γ_{cs} is

$$\Gamma_{cs} = \xi \frac{s_{cs}}{D} \qquad (3.37)$$

where s_{cs} is creep ultimate strain taken as 0.0003

$$\xi = 0.72 \frac{p - p'}{\sqrt{p}} \leq 1.0$$

$$\text{for } 0.25 \leq p < 1.0 \qquad (3.38)$$

$$= 0.65 \frac{p - p'}{\sqrt{p}} \leq 0$$

$$\text{for } p \geq 1.0 \qquad (3.39)$$

where p and p' are the percentages of tensile steel and compressive steel respectively

$$p = \frac{100 \ A_{st}}{bd} \qquad (3.40)$$

$$p' = \frac{100 \ A_{sc}}{bd} \qquad (3.41)$$

$$y_c = y_{ic} - y_i \qquad (3.42)$$

where y_{ic} is the creep deflection associated with the initial deflection due to permanent loads. The Young's modulus of elasticity is

$$E_{ce} = E_c/(1+\phi) \qquad (3.43)$$

where ϕ is the creep coefficient of concrete

y_i is creep deflection due to permanent loads.

3.3.3 LIMIT STATE OF LOCAL DAMAGE

The effect of local damage due to cracks in the

beams is elaborated in this work. The cracks in the reinforced concrete beams are random and the variations in their width is very large. There is not much of research work carried out to determine, accurately the widths of cracks in a beam. The width of cracks are determined by empirical formulae (10). The surface crack width in general is

$$w_{cr} = \frac{3\, a_{cr}\, s_m}{1+2\left[(a_{cr} - c_{min}) / (D-d_n)\right]} \qquad (3.44)$$

where a_{cr} = the distance from the nearest reinforcing bar to the point where the crack width is going to be calculated as

s_m = strain which is given by

$$s_m = s_1 - \frac{1.2\, b_t D(a'-d_n) \times 10^{-3}}{A_{st}(D-d_n)\, f_y} \qquad (3.45)$$

a' = the distance from the compressive face to the point of crack

s_1 = strain at level considered ignoring concrete intension zone

b_t = width of the section at the level of centre of gravity of tension steel

c_{min} = minimum of all values of a_{cr}.

These crack widths are limited to maximum values as given in the following chapter.

CHAPTER IV

FORMULATION OF OPTIMIZATION PROBLEM

4.1 INTRODUCTION

The invention of the digital computers has enabled engineers solve complex problems by numerical methods. Of late, various operations in structural design have been subject to extensive research, the most significant being the optimization of structures. A design does not mean simple proportioning of the relevent components of the structure but should give a structure which is optimum and efficient. Optimization of the most common element in structures namely the Tee beam has been attempted herein. There are number of mathematical methods which enable an engineer to optimize.

4.2 GENERAL FORMULATION AND METHODS OF SOLUTIONS

A structural system can be described by a set of quantities which are viewed as variables during the design process. The quantities that are fixed at the outset are called preassigned parameters, while those quanities that are arrived at during the process of design are called design variables.

The objective function in a structural design is the basis for choice amongst all the alternative designs, as the design has infinite number of solutions . The optimization of a total structure as a whole has bearing on the optimization of each element of the structure. It is difficult to select the objective function which effectively relates the element to the whole structure. So, one of the guiding factors to select the objective function may be the one which is an important design property that can possibly be quantified.

A general optimization problem can be stated as

find $\vec{D}$,

that minimizes an objective function

$$F(\vec{D}) \tag{4.1}$$

such that $l_k(\vec{D}) = 0$ for $k = 1,2,\ldots,k$ (4.2)

and $g_j(\vec{D}) \leq 0$ for $j = k+1, k+2,\ldots,m$ (4.3)

The design vector $\vec{D}$ is a point in n-dimensional design space, function $l_k(\vec{D})$ denotes the equality constraints and $g_j(\vec{D})$ the inequality constraints. The objective function is $F(\vec{D})$.

Many classical methods and numerical methods are available to seek solution of the problem stated in

equations (4.1), (4.2) and (4.3). The difficulty in solving the problem by classical methods necessitates the application of numerical techniques. There are algorithms like linear programming, non-linear programming, geometric programming, dynamic programming etc. One of the methods to solve such a problem is a linear programming method wherein the equations (4.1), (4.2) and (4.3) are suitably replaced by linear approximation by Taylor series expansion about the point $\vec{D}$, neglecting all the quadratic and higher order terms. This alternative enables the application of well established linear programming algorithms to solve the given basic problem.

There are other methods which can handle the problem as given in the preceding paragraph. The methods like penalty function formulations are widely used methods for nonlinear problems where the objective function and the constraints are nonlinear functions of the design variables.

4.2.1 PENALTY FUNCTION METHODS

Of the methods of optimization of a nonlinear constrained problem, penalty function methods are the most widely used. Penalty function method transforms the basic constrained optimization problem into alternative

formulations such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. The general form of formulation of an unconstrained minimization problem can be given as

$$\phi_k = f(\vec{D}) + r_k \sum_{i=1}^{m} G_i \left[l_k(\vec{D}) \;,\; g_j(\vec{D}) \right] \qquad (4.4)$$

where G_i is some function of equality and inequality constraints $l_k(\vec{D})$ and $g_j(\vec{D})$ respectively and r_k is the penalty parameter. If the unconstrained minimization of ϕ_k is repeated for a sequence of values of $r_k(k=1,2,3\ldots)$, the solution may be brought to converge to that of the original problem stated by equations (4.1), (4.2) and (4.3). This is the reason why penalty function methods are also known as sequential unconstrained minimization techniques (SUMT).

The algorithms available to adopt the technique of SUMT are many in number. They can be classified into board catagories as direct search methods and descent methods. The direct search methods do not require the partial derivatives of the given function to be optimized, while the descent methods need the gradients. For the same reason the direct search methods are also called nongradient methods. Comparatively descent methods are more efficient if the function consists of a large number

of variables. The descent method called 'Variable Metric method' or the 'Davidon Fletcher Powell' (DFP) method (14) is used in the present optimization. The DFP method is used along with cubic interpolation method for one-dimensional minimization, to obtain the step length λ_{opt} to determine the new design vector.

$$\vec{D}_{i+1} = \vec{D}_i + \lambda_{opt} \vec{S}_i \qquad (4.5)$$

The interior penalty method is used in the penalty methods to solve the constrained problem. The main advantage of this being that the vector obtained after every SUMT is in the feasible domain of design space. But it is necessary that this method has to be supplied with an initial vector $\vec{D}_0$ which is in feasible domain of design space. In case of engineering problems this does not pose a great problem as it is not much very difficult to supply a $\vec{D}_0$ which is in the feasible domain. There is also a method to obtain an initial feasible point using the same penalty function but it is out of scope of the present work. The formulation in case of the present problem is

$$\phi_k = F(\vec{D}) - r_k \sum_{i=1}^{m} \frac{1}{g_j(\vec{D})} \qquad (4.6)$$

as there are no equality constraints.

4.3 FORMULATION OF THE PROBLEM

In many practical problems, the design variables can not be choosen arbitrarily; rather they have to satisfy certain specified functional and other requirements. The restrictions that must be satisfied in order to produce an acceptable design are collectively called design constraints. The constraints which represent limitations on the behaviour or performence of the system are called behaviour or functional constraints. The constraints which represent physical limitations on the variables are called geometric or side constraints. With the foregoing in view the constraints are divided into two groups and are given as follows.

4.3.1 BEHAVIOUR CONSTRAINTS

The constraints that are to be considered in this problem are listed in the following.

(a) The limit state of moment (M_s) in the slab due to the physical influence should be less than or equal to the moment of resistance (M_{rs}) of the slab section so that the slab is safe against flexural failure.

$$M_s \leq M_{rs} \tag{4.7}$$

(b) The Tee beam should have sufficient flexural strength to resist the limit state moment (M_B) caused due to the external loads

$$M_B \leq M_{rB} \qquad (4.8)$$

(c) The total shear force should safely be resisted by the shear capacity of the beam

$$v_B \leq v_{all} \qquad (4.9)$$

(d) The beam should have sufficient stiffness associated with the flexural strength so as to be functionally sound and should limit the deflection (y_1, y_2) to a maximum of span (l_b)/250 in case of total load and to span (l_b)/350 in case there is the dead load plus the permanent load. The deflection is the total of all due to shrinkage, creep, temperature including the initial elastic deflection.

$$y_1 \leq (y_{all})_1 \qquad (4.10)$$

$$\text{where } (y_{all})_1 = l_b/250 \qquad (4.11)$$

$$y_2 \leq (y_{all})_2 \qquad (4.12)$$

$$\text{where } (y_{all})_2 = l_b/350 \text{ or } 20 \text{ mm}$$
$$\text{whichever is less} \qquad (4.13)$$

(e) It is also necessary that the tensile cracks in concrete in the tension zone are limited to

account for the functional utility of the structure and to safeguard against corrosion of steel and spalling of concrete. The maximum surface crack width (w_m) is limited to 0.3 mm and the surface crack width (w_s) just below the reinforcing bar to 0.4 d_s

$$w_m \leq 0.3 \tag{4.14}$$

$$w_s \leq 0.4\, d_s \tag{4.15}$$

(f) To avoid sudden failure of concrete and to impart ductility to the structure the depth of neutral axis (d_n) is limited at the most to the balanced or a limiting depth of neutral axis (d_{nl})

$$d_{nl} = \frac{-\,0.0715}{250} f_{cu} + 0.6 \tag{4.16}$$

$$d_n \leq d_{nl} \tag{4.17}$$

(g) The local bond stress should be less than or equal to the allowable bond stress to have no slip condition

$$v_b \leq v_{ba} \tag{4.18}$$

4.3.2 GEOMETRIC CONSTRAINTS

(h) For all practical conditions the code spefifies a limit on the depth of the slab, which in turn is the depth of the flange, to 75 mm.

$$75 \leq d_f \tag{4.19}$$

(i) To place the reinforcement effectively the width of the rib of the Tee beam has to be limited to 150 mm or so.

$$150 \leq b \qquad (4.20)$$

(j) Stiffness limitations on the slab as specified by code . (11) limits the span (l_s) to depth (d_s) ratio of the slab to 26.

$$l_s/d_f \leq 26 \qquad (4.21)$$

(k) The span of the beam (l_b) is arbitrarily limited to 10,000 mm as it is uneconomical to have larger spans.

$$l_b \leq 10000 \qquad (4.22)$$

(l) The code specifies the limitation on span (l_b) to depth (d) ratio to control the deflection of the beam. As rigorous analysis is done the span to depth ratio is arbitrarily limited to a lesser value than that of the code to make this constraint an inactive constraint.

$$l_b/d \leq D_f \qquad (4.23)$$

where D_f = 10 for cantilever beam

= 15 for simply supported beam

= 20 for continuous beam (4.24)

(m) To take care of temperature effects and to control the cracks there is a minimum amount of longitudinal

(A_{stm}) and shear reinforcement (A_{svm}) to be provided.

$$A_{stm} \leq A_{st} \quad (4.25)$$

$$A_{svm} \leq A_{sv} \quad (4.26)$$

where $A_{stm} = 0.85\ bd/f_y$ (4.27)

$$A_{svm} = 0.002\ b\ c_v \quad (4.28)$$

c_v is the spacing of shear links.

(n) The spacing of links should be so that every potential crack is crossed by a link. To ensure this a maximum spacing of 0.75d is stipulated

$$c_v \leq 0.75\ d \quad (4.29)$$

(o) In no case a design variables are less than zero as they are all physical quantities

$$-x_i \leq 0 \quad (4.30)$$

$$i=1,2,3,\ldots,11$$

4.3.3 THE DESIGN VECTOR AND THE COST FUNCTION

The design vector is

$$\vec{D} = (x_i | i=1,2,3,\ldots,n) \quad (4.31)$$

where n is, the number of design variables, equal to 11 in the present problem.

Refering to Fig.(3.3) the components of the design vector are

$$x_1 = B \qquad x_4 = d \qquad x_7 = A_{sc}$$
$$x_2 = b \qquad x_5 = D \qquad x_8 = A_{sv}$$
$$x_3 = d_f \qquad x_6 = A_{st}(\text{beam}) \qquad x_9 = A_{st}(\text{slab})$$
$$x_{10} = c_{st}$$
$$x_{11} = c_v \tag{4.32}$$

where all terms are as defined earlier.

The foregoing vector is to be determined subject to the constraints given in articles 4.3.1 and 4.3.2, optimizing the cost function or the so called objective function.

$$\text{Total cost} = F(\vec{D}) = C_c \left[Bd_f + b\,(D-d_f)\right] l_b + (A_{sc} + A_{st} + A_{sr})\, l_b\, R_1\, C_c + (l_b + 2D - 2d_f)\, R_2\, l_s\, C_c/l_b \tag{4.33}$$

Nondimensionalizing it

$$\text{Cost}/C_c = B\,d_f + b(D-d_f)\;\; l_b + (A_{sc}+A_{st}+A_{sv})\, l_b\, R_1 + (l_b+2D-2d_f)\, R_2\, l_s/l_b \tag{4.34}$$

where $$R_1 = r_s\, l_s/l_c \tag{4.35}$$

r_s is the density of steel N/mm^3 and C_s is the unit cost of steel Rs/Newton.

$$R_2 = \frac{C_w\, l_b}{C_c} \tag{4.36}$$

C_w is the unit cost of wood Rs/mm^2.

4.3.4 FORMULATION-STANDARD FORM

Find vector $\vec{D}$ $(x_i | i=1,2,3,\ldots,11)$ to minimize

$$F(\vec{D}) = \left[B\, d_f + d\,(D-d_f)\right] l_b$$
$$+ (A_{sc} + A_{st} + A_{sv})\, l_b\, R_1$$
$$+ (l_b + 2D - 2d_f)\, R_2\, l_s/l_b \qquad (4.37)$$

subject to

$$g_1 = M_s / M_{rs} - 1 \leq 0$$
$$g_2 = M_B / M_{rB} - 1 \leq 0$$
$$g_3 = v_B / v_{all} - 1 \leq 0$$
$$g_4 = y_1 / (y_{all})_1 - 1 \leq 0$$
$$g_5 = y_2 / (y_{all})_2 - 1 \leq 0$$
$$g_6 = W_m / 0.3 - 1 \leq 0$$
$$g_7 = W_s / 0.4\, d_s - 1 \leq 0$$
$$g_8 = d_n / d_{nl} - 1 \leq 0$$
$$g_9 = v_b / v_{ba} - 1 \leq 0$$
$$g_{10} = 75/d_f - 1 \leq 0$$
$$g_{11} = 150 / b - 1 \leq 0$$
$$g_{12} = l_s / 26\, d_f - 1 \leq 0$$
$$g_{13} = l_b / 10000 - 1 \leq 0$$
$$g_{14} = l_b / D_f d - 1 \leq 0$$
$$g_{15} = A_{stm}/A_{st} - 1 \leq 0$$
$$g_{16} = A_{svm}/A_{sv} - 1 \leq 0$$
$$g_{17} = c_v / 0.75\, d - 1 \leq 0$$

$$g_j = - x_i \leq 0$$

for $i=1,2,3,\ldots,11$

and $j=18,19,\ldots,27$ (4.38)

The problem stated in the foregoing equations (4.37) and (4.38) is solved by interior penalty function method. The results and discussion are presented in the following chapters.

cost of steel is assumed to be Rs. 3.50, 4.00 or 4.50 per kilogram. This accounts for the fluctuations in the costs of these vital materials of construction. The variation in the cost of form work is not taken into account as the type of material choosen for form work differs greatly from place to place and usually the cheapest material is used. A constant value of Rs.25 per square meter is assumed as the cost of form work. It is fairly easy to incorporate the variation in the cost of form work if sufficient data is available. A set of nine problems are solved for given spans of the beam and the slab. The cost ratio (CR) versus the total cost, per millimeter span of the beam, for different span ratios are plotted as in Figs. 5.1 through 5.7. The cost ratio is calculated as :

$$CR = \rho_s \frac{C_s}{C_c} \qquad (5.1)$$

where ρ_s is the density of steel, C_s and C_c are the costs of steel and concrete respectively. It can be observed that different curves result in for different costs of steel.

The span ratio between the beam and the slab is given by

$$\text{Span ratio} = l_b/l_s \qquad (5.2)$$

Table 5.1 is drawn giving the areas of steel for a given span of slab (3000 mm), with different spans of the

beam. The costs of concrete and steel of Rs. 400 per cubic meter and Rs. 4 per kilogram respectively, are adopted for the optimization of the area of steel in the Table 5.1. Table 5.2 gives the areas of steel for a single span ratio and different cost ratios.

TABLE 5.1 : AREAS OF STEEL FOR DIFFERENT SPANS OF THE BEAM FOR 3000 MM SPAN OF THE SLAB

S.No.	Span of the beam (mm)	Area of tension steel (mm^2)	Area of compressive steel (mm^2)	Area of shear reinforcement (mm^2)
1	3600	7995.57	0.0	990.17
2	4000	7995.57	0.0	990.17
3	4500	7995.57	0.0	990.17
4	5000	7995.57	0.0	949.00
5	6000	7996.13	0.0	1092.68
6	7000	7996.13	0.0	1092.68
7	8000	7996.13	0.0	1092.68
8	9000	7996.13	0.0	1092.68

TABLE 5.2 : AREAS OF STEEL FOR DIFFERENT COST RATIOS

THE SPAN RATIO IS 2.5

S.No.	Cost of concrete (Rs/m^3)	Cost of Steel (Rs/kg)	Area of tension steel (mm^2)	Area of compressive steel (mm^2)	Area of shear reinforcement (mm^2)
1	400.0	3.50	7989.75	0.0	774.82
2	500.0	3.50	7989.75	0.0	774.82
3	600.0	3.50	7989.75	0.0	774.82
4	400.0	4.00	7989.75	0.0	774.82
5	500.0	4.00	7989.75	0.0	774.82
6	600.0	4.00	7989.75	0.0	774.82
7	400.0	4.50	7989.75	0.0	774.82
8	500.0	4.50	7989.75	0.0	774.82
9	600.0	4.50	7989.75	0.0	774.82

It is interesting to note that the area of steel is almost the same in all the cases. This can be visualised by studying the results presented in Table 5.1 and Table 5.2.

The initial feasible point given and the optimal design vector obtained are shown in Table 5.3 for a typical example with span ratio of 2.833. It is observed that the initial feasible point is close to the optimal point. Of this reason, the problem has converged very fast and at the

most it took three cycles of SUMT with the initial value of the penalty parameter $r_k = 1.0$.

TABLE 5.3 : TABLE SHOWING INITIAL FEASIBLE POINT AND THE OPTIMAL DESIGN VECTOR (SAPN RATIO=2.833)

Design Vector	Initial feasible point $\vec{D}_o$	Optimal design vector $\vec{D}_{opt}$
x_1	0.0 mm	1124.96 mm
x_2	200.0 mm	151.71 mm
x_3	200.0 mm	115.54 mm
x_4	950.0 mm	1219.45 mm
x_5	990.0 mm	1259.45 mm
x_6	8000.0 mm^2	7989.86 mm^2
x_7	0.0 mm^2	0.0 mm^2
x_8	150.0 mm^2	722.82 mm^2
x_9	20.0 mm^2	683.12 mm^2
x_{10}	50.0 mm	668.20 mm
x_{11}	200.0 mm	461.32 mm

The same initial point is choosen for all span ratios. But the obtained optimum point is successively used as the initial point for other cost ratios.

5.3 CONCLUSIONS

(1) The hierarchy of the values of the critically satisfied constraints in general is observed to be in the order as follows:

(i) the limit of depth of natural axis, tending the design to be a balanced design,

(ii) the limit on the span to depth ratio of slab,

(iii) the limit on the depth of the slab,

(iv) the limit on the minimum width of the rib,

(v) the limit on the spacing of the shear reinforcement,

(vi) the limit on the span to depth ratio of the beam,

(vii) the moment capacity of the slab.,

(viii) the deflection y_2 of the beam.,

(ix) the limit on the minimum area of shear reinforcement,

(x) the moment capacity of the beam,

(xi) the limit on the deflection y_1 of the beam,

(xii) the maximum crack width,

(xiii) the average crack width and

(xiv) the shear capacity of the beam.

The constraints specifying the nonnegativity of the variables are neglected. It is also observed that almost all the examples followed the same hierarchy. All the constraints that are critically satisfied are the geometric constraints. This infers that the beam is sound in its behaviour. Hence,

the codes can be relexed in constraining the geometry of the structural elements in ordinary buildings where there is not heavy loading.

(2) As the computed deflections are less than the permissible deflections the span to depth ratio in the codes can be very well relaxed as the limitation on the span to depth ratio on the beam is arbitrarily fixed in the examples under consideration. It can also be concluded that the optimal beams have high flexural rigidity.

(3) As there is no limit prescribed on the maximum depth of the beam the optimal solutions will have ribs of large depths. Now-a-days architects tend to adopt diaphram type ribs in case of beams. It is so observed that all the solutions giving optimal design for cost invariably have given diaphram ribs. But a rigorous analysis is to be done for the lateral buckling and the effect of shear on the deep beam.

(4) Variation in costs is assumed for concrete and steel and the curves indicated that the total cost as a function of the span of the beam is less when the cost ratio is larger. For given costs of steel and concrete the total cost of the beam can easily be computed from the graphs. As an example let us assume the market price of steel to be Rs.4.0 per kilogram and that of concrete to be Rs. 400.0 per cubic meter. Assuming the density of steel to be 7.85 gm/cc, the

cost ratio CR is

$$CR = \frac{0.0078 \times 40}{400 \times 10^{-9}} = 780.0 \qquad (5.5)$$

where the quantities are expressed in terms of Newtons and millimeters in the equation (5.5).

Then, the total cost of structure can be obtained for a given span ratio for the optimal problem. Assume a span ratio of 2.5. Hence the total cost of the structure from Fig.5.5 can be obtained as 0.22 per millimeter span of the beam. Knowing the span of the beam in millimeters say 5000 mm, we can calculate the total cost as

$$\begin{aligned} \text{Total cost} &= 0.22 \times 5000 \text{ Rs.} \\ &= \text{Rs. } 1100.0 \end{aligned} \qquad (5.6)$$

(5) Observing the hierarchy of the values of constraints, the more critical being the geometric constraints, it can be concluded that the codes are still conservative and a modification can always be incorporated. But it should be borne in mind that a complete check for slenderness is yet to be done.

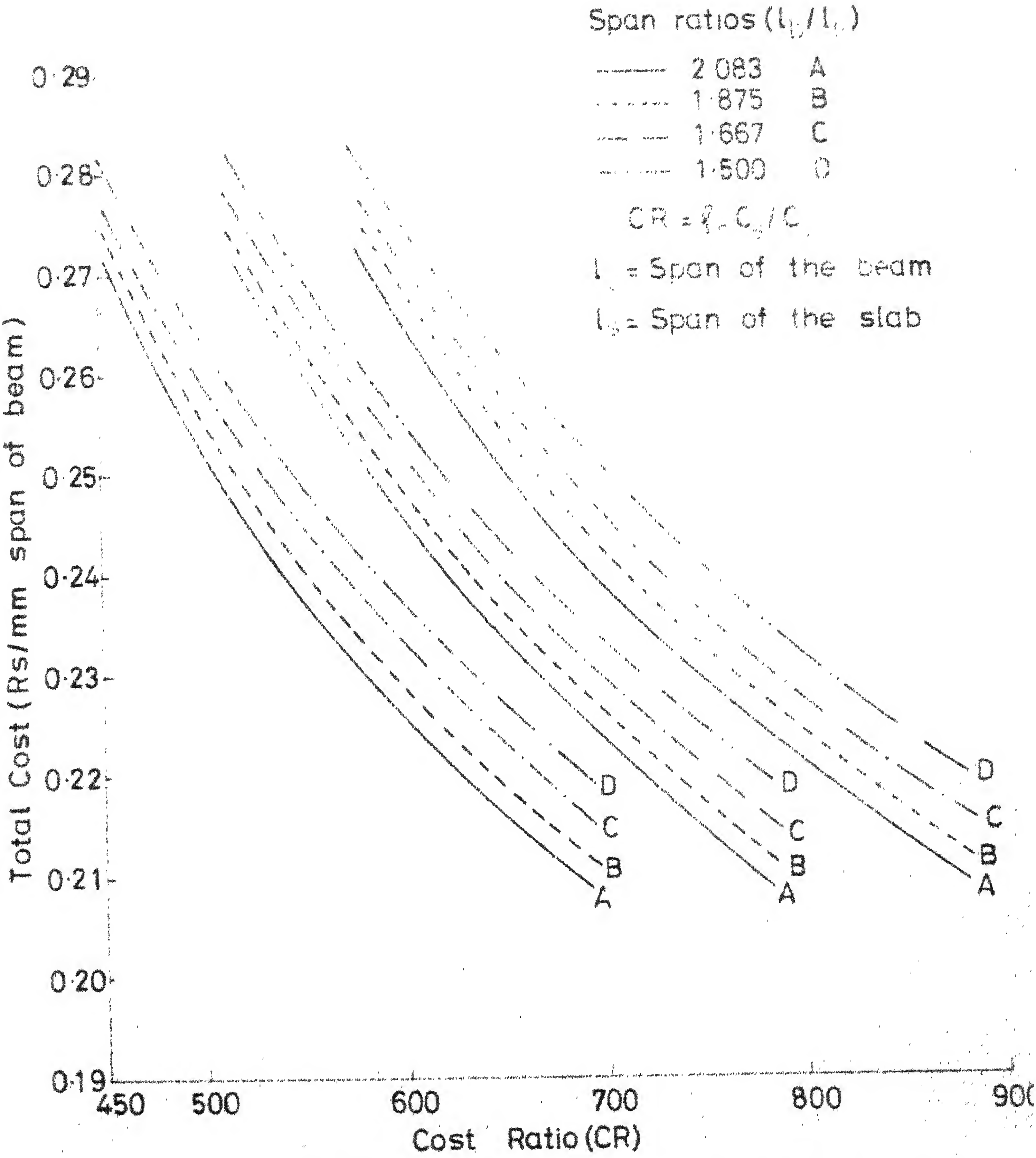

Fig. 5·1 Cost Ratio vs Total Cost

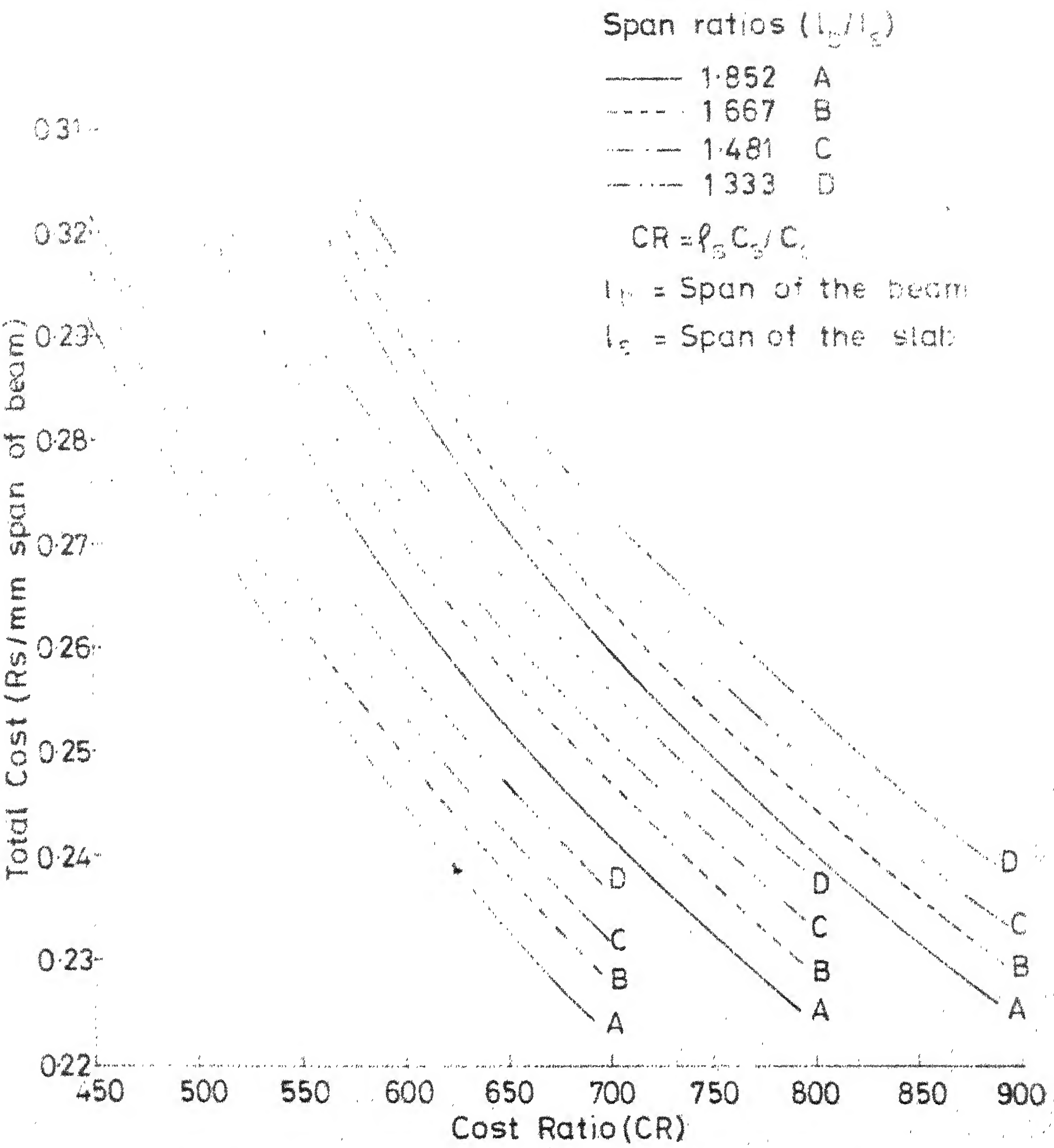

Fig. 5·2 Cost Ratio vs Total Cost

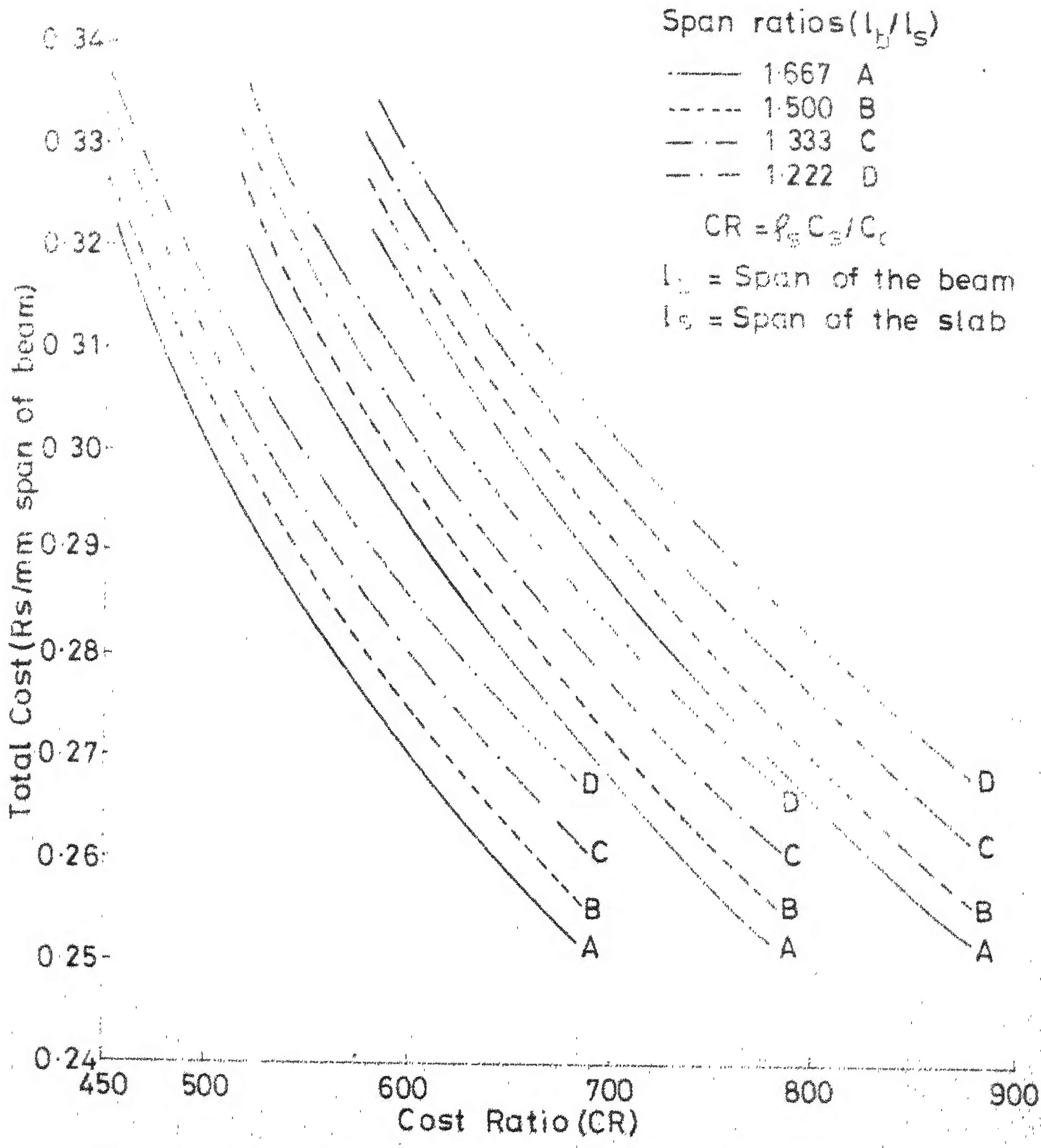

Fig. 5·3 Cost Ratio vs Total Cost

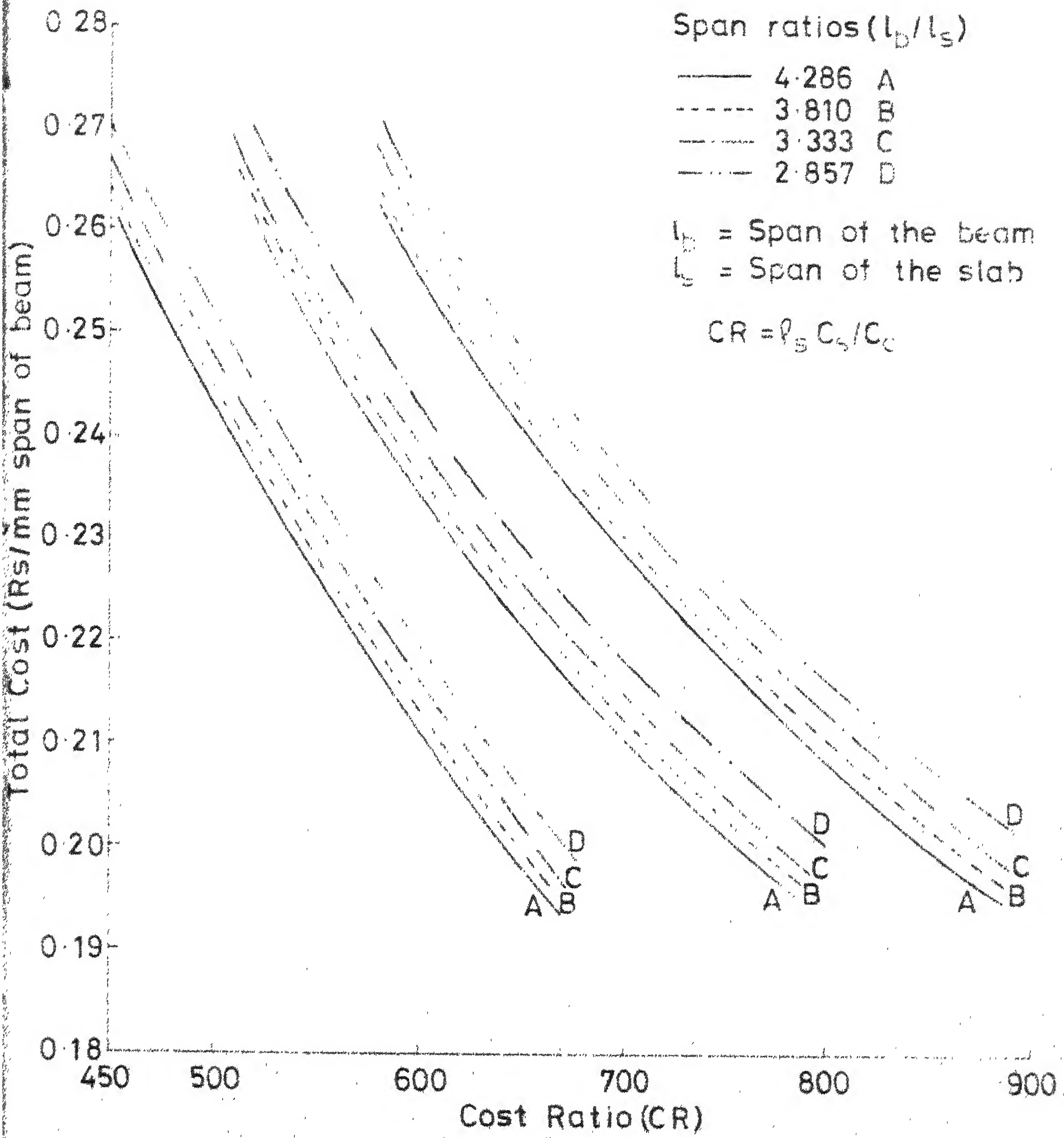

Fig. 5·4 Cost Ratio vs Total Cost

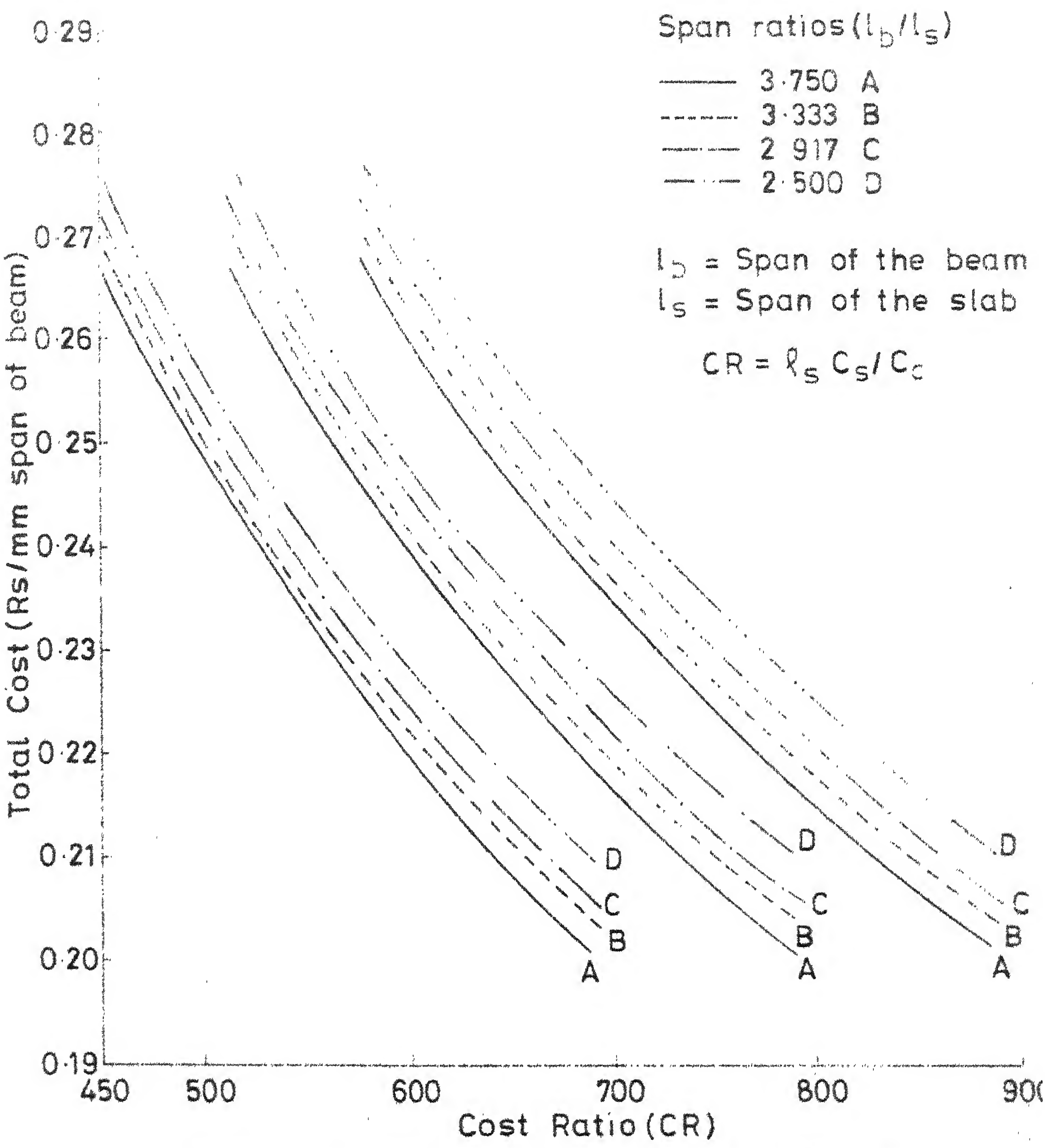

Fig. 5·5 Cost Ratio vs Total Cost

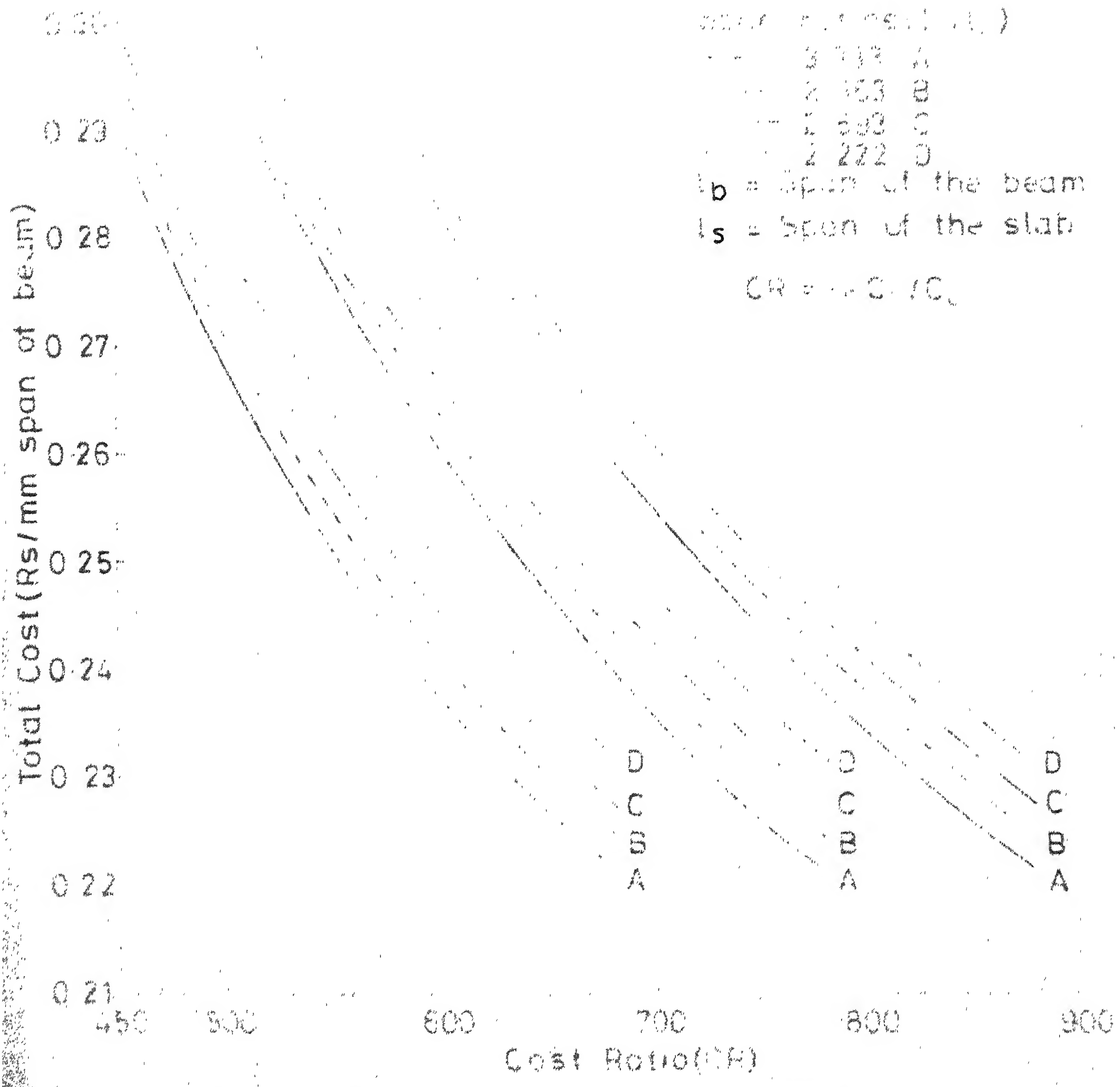

Fig 5 6 Cost Ratio vs Total Cost

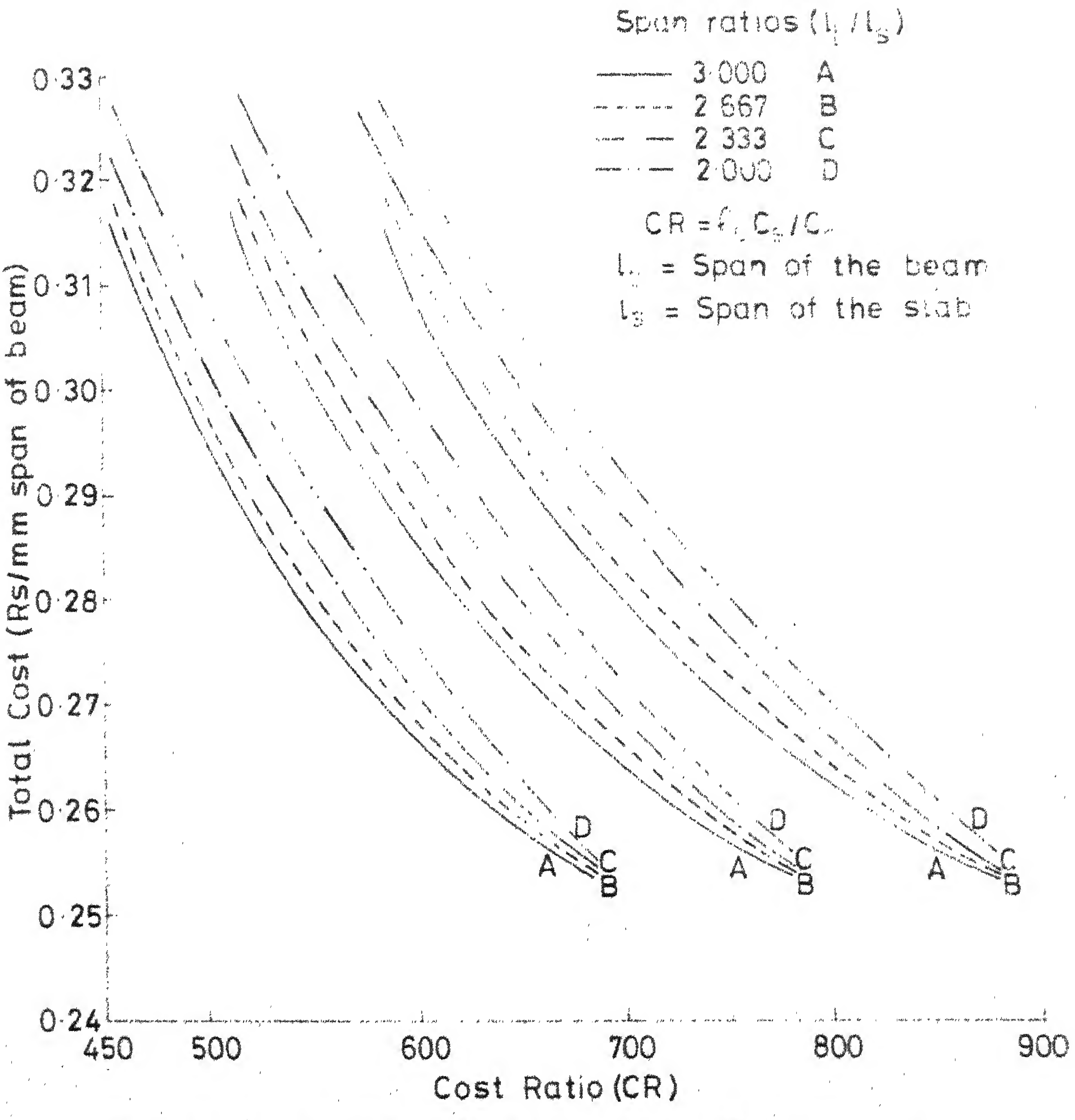

Fig 5·7 Cost Ratio vs Total Cost

CHAPTER VI

OVER VIEW

6.1 GENERAL

The traditional methods that are in use are not rational regarding the actual factor of safety. It is a well known fact that the actual factor of safety in case of the well established elastic method of design varies with the loading and structure. The plastic methods developed later have considered this factor and have suggested load factors to compute the design loads. Even then these plastic methods have not reflected any light about the structural behaviour at working loads. But it is absolutely necessary for a structure to be rationally designed and checked for all serviceability conditions. So a method called the "Limit state method" is the culmination of all the methods evolved hitherto. It incorporates the load factor design with due account of analysis of the structure for working loads. This method of design has been taken up and a Tee beam element is optimized. This is a step in the right direction of optimizing large systems. The present method of analysis

can further be improved.

6.2 SUGGESTIONS FOR FUTURE WORK :

1. The same type of work may be repeated varying the grade of concrete and the nature of steel reinforcement to get exhaustive results for various types of reinforcement and grades.

2. The rectangular parabolic stress block can further be improved by incorporating Hogenstad's (1) stress block.

3. The member can be analysed as a continuous member, with due consideration of the different moments of inertia at the midspan and supports.

4. The redistribution of the bending moments and different flexural rigidities have to be incorporated in case of continuous members.

5. The bilinear nature of the moment curvature relation should be accounted for computing the deflections of the structure with uncracked and cracked section.

6. The total structural system may be optimized along with the elements of it and study of the

cost in both the cases should be studied.

7. A detailed analysis with different ways of reinforcement in the estimation of crack widths, can be carried out.

8. The study may be extended with the application of probability theories.

9. Optimization may be done with different methods of programming like Geometric programming, Dynamic programming etc. and the results be compared for further insight.

REFERENCES

1. Barry P. Hughes; "Limit state theory for reinforced concrete", Pilmon Publishing, 1971.
2. Bengt B. Broms; "Crack width and spacing in R.C. members"; Jl. Amer. Conc. Inst., V 62, 1965, pp.1237-1257.
3. ACI Committee Report; "Control of cracking in concrete structures"; Jl. Amer. Conc. Inst., V 69/12, 1972, pp 717-754.
4. Peter Gergley; "Distribution of reinforcement for crack control"; Jl. Amer. Conc. Inst.
5. Edward G. Nawy; "Crack control through reinforcement distribution in two-way acting slabs and plates"; Jl. Amer. Conc. Inst.; V 69/4, Apr. 1972, pp. 217-220.
6. Glen S. Orenstein and Edward G. Nawy "Crack width control in R.C. two-way slabs subjected to a uniformly distributed load"; Jl. Amer. Conc. Inst. V 67, 1970, pp. 57-61.
7. Robert H. Elvery and Mohamad Shafi; "Analysis of shrinkage effects on R.C. structures"; Jl. Amer. Inst., V 67, 1970, pp. 45-53.
8. Dan E. Brauson; "Deflections of R.C. flexural members", Jl. Amer. Conc. Inst.; V 63/1; Jan 1966, pp. 637-674.

9. Donald G. Alcock and Andrian Pauw; "Controlled deflection design method for R.C. beams and slabs"; Jl. Amer. Conc. Inst.; V 59, May 1962, pp. 645-659.

10. T.J. MacGinly; "Reinforced Concrete-Design theory and examples"; E & F.N. Spon Ltd., 1978.

11. CP 110 : 1975; "Code of Practice for structural use of concrete"; 1975.

12. Cement and Concrete Association; "Handbook on the Unified Code for structural concrete" (CP 110:1972); 1974.

13. IS : 456 (Draft Code) : Code of practice (Third revision) for reinforced concrete structures, 1978.

14. Rao, S.S; "Optimization - theory and applications"; Wiley Eastern Limited, 1978.

15. G.I.N. Rozvany; "Optimal design of flexural systems"; The Pegamon Press; 1976.